RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. FOURTH SEMESTER EXAMINATION, MAY 2017

SECOND YEAR [BATCH 2015-18]

Date : 23/05/2017 **MATHEMATICS (General)**

Time : 11 am – 2 pm **Paper :** IV Full Marks : 75

[Use a separate Answer Book for each group]

Group - A

Answer **any four** questions from **Question Nos. 1 to 6** : [4×5]

1. a) Evaluate : $\int_0^3 \frac{dx}{\sqrt{9-x^2}}$. [3]

   b) Find the value of $\Gamma\left(\frac{9}{2}\right)$. [2]

2. Evaluate $\iint_R \sqrt{x^2+y^2}\,dx\,dy$, where R is the region bounded by the circles $x^2+y^2=1$ and $x^2+y^2=4$. [5]

3. Test the convergence of $\int_0^{\pi/2} \frac{x^m}{\sin^n x}dx$, n > m. [5]

4. Find the length of the curve $x=e^\theta \sin\theta$, $y=e^\theta\cos\theta$ between the points $\theta=0$ and $\theta=\frac{\pi}{2}$. [5]

5. Prove that the surface area of the solid formed by revolving the cardioid $r=a(1+\cos\theta)$ about the initial line is $\frac{32\pi}{5}a^2$. [5]

6. a) Show that $B(m,n)=B(m+1,n)+B(m,n+1)$. [3]

   b) Discuss the convergence of $\int_1^\infty \frac{dx}{x\log x}$. [2]

Answer **any two** questions from **Question Nos. 7 to 9** : [2×5]

7. Solve : $x^2\frac{d^2y}{dx^2}-x\frac{dy}{dx}+y=\log x$. [5]

8. Solve $(D^2-1)y=x\sin x+(1+x^2)e^x$. [5]

9. Find the orthogonal trajectories for the following family of curves:

   a) $y^2=4x^2(1-ax)$, a being variable parameter. [2]

   b) $r=2a(\sin\theta+\cos\theta)$, a being variable parameter. [3]

Group - B

Answer **any three** questions from **Question Nos. 10 to 14** : [3×15]

10. a) State and prove Bayes' Theorem on n pair-wise mutually exclusive events each with positive probability. [2+4]

    b) There are two identical urns containing 3 white and 4 black balls; 7 white and 3 black balls. An urn is chosen at random and a ball is drawn from it. Find the probability that the ball is black. What is the probability that it is from the first urn if the ball drawn is black? [3+2]

    c) If events A^C and B are independent, show that so are (i) A and B^C, (ii) A^C and B^C. [2+2]

11. a) Determine the value of the constant k such that f(x), defined by

$$f(x) = \begin{cases} kx(1-x) & , \quad 0 < x < 1 \\ 0 & , \quad \text{elsewhere} \end{cases}$$

    is a probability density function. Find the corresponding distribution function and $P\left(X > \frac{1}{3}\right)$. [2+5+3]

    b) The joint pdf of random variables X and Y is given by

$$f(x,y) = \begin{cases} kxy & , \quad 0 < x, y < 1 \\ 0 & , \quad \text{otherwise} \end{cases}.$$

    Find the constant k and check if X, Y are independent. [2+3]

12. a) A radioactive source emits on the average 4·5 particles per second. Calculate the probability that 3 or more particles will be emitted in an interval of 4 seconds. [5]

    b) The frequency distribution of expenditure of 1000 families is

Expenditure (Rs.)	40 – 59	60 – 79	80 – 99	100 – 119	120 – 139
Number of families	50	?	500	?	50

    The mean and median of the distribution are both Rs. 87·50. Determine the missing frequencies. [5]

    c) The radius X of a circle has uniform distribution in (1,2). Find the mean and variance of the area of the circle. [5]

13. a) Using three-yearly working averages, find the trend values for the following series [3]

Year	1	2	3	4	5	6	7
Value	2	4	5	7	8	10	13

    b) Determine the equation of a straight line which best fits the following data : [4]

Year	2001	2002	2003	2004	2005
Sales (in Rs.)	35	56	79	80	40

    c) Find by the weighted Aggregative Method, the index number from the following table : [4]

Commodity	Base (2001) Price (per unit)	Current (2005) Price (per unit)	weight (kg)
Rice	320	500	8
Wheat	250	250	6
Oil (edible)	900	1000	7
Fish	1200	1400	3
Potato	350	400	5

d) The following data are the consumer price indices for 5 groups with percentages of total expenditure for middle class people of a certain city in 2010 with 2000 as base year. Hence determine the consumer price index number of 2010 with 2000 as base. [4]

Group	Group Indices (%)	Percentage of total expenditure
Food	525	40
Clothing	325	16
Fuel & light	240	15
House rent	180	20
others	200	9

Mr. X got a salary of Rs. 55,000 in 2000. Determine how much he would have to receive as salary in 2010 to maintain his same standard of living as in 2000?

14. A printing inks manufacturer advertises that 1 unit of their ink can be used to print on an average 5000 A4 sheets of paper. A random sample of 40 units show the following data :

Unit No.	No. of sheets	Unit No.	No. of sheets	Unit No.	No. of sheets	Unit No.	No. of sheets
1	5000	11	4998	21	4998	31	5000
2	4990	12	4999	22	5000	32	5001
3	4993	13	4998	23	5001	33	4990
4	4996	14	4997	24	5003	34	4995
5	5002	15	5001	25	5005	35	4998
6	5010	16	5003	26	4996	36	5000
7	5003	17	5007	27	4998	37	4995
8	4993	18	4987	28	5001	38	5001
9	4992	19	4990	29	4993	39	4995
10	4990	20	5000	30	5001	40	5000

a) Calculate the mean and standard deviation of the above sample. [8]

b) Test the manufacturer's claim, given that 1% significance level is obtained at x = 2·576. [7]

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